Information-Driven Distributed Sensing for Efficient Bayesian Inference in Internet of Things Systems

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Overview

1. Introduction to Distributed Sensing in IoT
2. System Model
3. Problem Formulation
4. Solution Approaches
5. Theoretical Analysis
6. Performance Evaluation
7. Conclusions
Sensing agents (SAs) collaboratively estimate a global parameter $\theta$.

At certain time instants, only a subset of SAs may have valuable information.
Bayesian Inference and Exponential Family

Let \( p(x|\theta) \) denote the probability distribution of random variables \( x \in \mathbb{R}^m \), given parameters \( \theta \in \mathbb{R}^d \).

Assume \( p(x|\theta) \) is an exponential family distribution:

\[
p(x|\theta) = \exp\{\theta^T \phi(x) - A(\theta)\}.
\]

The conjugate prior of the model parameter \( \theta \):

\[
p(\theta|\omega, \nu) = \exp\{\theta^T \omega - A(\theta)\nu - \Lambda(\omega, \nu)\}.
\]

Posterior based in some measurements \( z = \{z_i\}_{i=1}^n \):

\[
p(\theta|z, \omega, \nu) = \exp\{\theta^T(\omega + \sum \phi(z))

- A(\theta)(\nu + n) - \Lambda(\omega + \sum \phi(z), \nu + n)\}.
\]
Define $\tilde{\omega} = [\omega^T, \nu]^T$, $\tilde{\theta} = [\theta^T, -A(\theta)]$ and $\tilde{\phi}(z) = [(\sum \phi(z_i))^T, n]^T$. Prior distribution on the model parameter $\theta$:

$$p(\theta|\tilde{\omega}) = \exp\{\tilde{\theta}^T \tilde{\omega} - \Lambda(\tilde{\omega})\}, \quad (4)$$

Posterior distribution on the model parameter $\theta$:

$$p(\theta|z, \tilde{\omega}) = \exp\{\tilde{\theta}^T (\tilde{\omega} + \tilde{\phi}(z)) - \Lambda(\tilde{\omega} + \tilde{\phi}(z))\}. \quad (5)$$

Additive update in the hyperparameters:

$$\tilde{\omega} \leftarrow \tilde{\omega} + \tilde{\phi}(z). \quad (6)$$
In this paper, we use the *KL divergence* between the prior distribution $p(\theta|\tilde{\omega})$ and the posterior distribution $p(\theta|z, \tilde{\omega})$ to quantify the information carried by the observations $z$:

$$u(z) = D_{KL}(p(\theta|\tilde{\omega})||p(\theta|z, \tilde{\omega})) = \Lambda(\tilde{\omega} + \phi(z)) - \Lambda(\tilde{\omega}) - \phi(z)^T \nabla \Lambda(\tilde{\omega}),$$

where $\nabla$ represents the gradient.
System Model

Assume we have a set of $\mathcal{N} = \{1, 2, \ldots, N\}$ distributed sensing agents (SAs).

Let $s_i(t) \in S = \{0, 1, \ldots, |S| - 1\}$ denote the channel state of SA $i$ for uploading sensing data in time slot $t$.

Sensing action of SA $i$: $a_i(t) \in \{0, 1\}$.

Energy cost model for SA $i$:

$$p_i(t) = \hat{p}_i(a_i(t), s_i(t)) = a_i(t) \frac{\alpha}{s_i(t) + \beta}. \quad (8)$$

The total energy consumption of the system in time slot $t$:

$$p(t) = \sum_{i \in \mathcal{N}} p_i(t). \quad (9)$$
The energy-constrained information utility maximization (EC-IUM) problem:

\[
\max: \quad \bar{u} = \lim_{T \to \infty} \frac{1}{T} \sum_{\tau=0}^{T-1} \mathbb{E}[u(\tau)] \quad (10)
\]

s.t.:
\[
\bar{p} = \lim_{T \to \infty} \frac{1}{T} \sum_{\tau=0}^{T-1} \mathbb{E}[p(\tau)] \leq C \quad (11)
\]

where \( C \) is a real number which specifies the constraint on the time average total energy consumption.
Centralized Solution (C-IDDS)

Virtual Queue for Energy Consumption Constraint in (11):

\[ Q(t + 1) = \max[Q(t) + p(t) - C, 0]. \tag{12} \]

Lemma

The constraint in (11) is satisfied if virtual queue \( Q(t) \) with backlog in (12) is stable.
Centralized Solution (C-IDDS)

Define the *Lyapunov function* for the defined virtual queue:

\[ L(t) = \frac{1}{2} Q(t)^2. \quad (13) \]

Define *one-slot Lyapunov drift*:

\[ \Delta(Q(t)) = L(t + 1) - L(t). \quad (14) \]

Reformulating the EC-IUM problem as another problem of minimizing the *upper bound* of the *drift-minus-utility*:

\[ \min \sup \Delta(Q(t)) - V \mathbb{E} \{ u(t) | Q(t) \}, \quad (15) \]

where \( V \geq 0 \) is a control parameter for gauging the trade-off between the utility and convergence rate.
Derive Upper Bound of the Drift-minus-utility Function

Upper Bound of the Drift-minus-utility Function:

Lemma

Under arbitrary control decisions, the following inequality holds

\[ \Delta(Q(t)) - \mathbb{E}\{u(t)|Q(t)\} \leq B - CQ(t) + Q(t)p(t) - Vu(t) \] (16)

where \( B = \frac{1}{2} C^2 \) is a finite constant.

Thus,

\[ (C-IDDS) \quad \min_{a_i(t), \forall i \in \mathcal{N}} Q(t) \sum_{i \in \mathcal{N}} a_i(t) \frac{\alpha}{s_i(t) + \beta} - Vu(t) \] (17)

s.t. \( a_i(t) \in \{0, 1\}, \forall i \in \mathcal{N} \). (18)
Remark.

- Require strict real-time communication with SAs.
- Non-scalable when the number of SAs become large.
**Goal:** SAs make sensing decisions and upload the sensing data *independently* based on their own states.
Charactering Strategy Space

**Goal:** SAs make sensing decisions and upload the sensing data *independently* based on their own states.

Define a *distributed decision rule*:
\[ \hat{a}(s(t)) = \{ \hat{a}_1(s_1(t)), \hat{a}_2(s_2(t)), \ldots, \hat{a}_N(s_N(t)) \}, \]
where \( a_i(t) = \hat{a}_i(s_i(t)) \).

Total number of possible strategies:
\[ M = \prod_{i=1}^{N} 2^{|S|}. \]

Denote \( \hat{a}^{(m)}(s) \) as a particular distributed strategy among the all \( M \)
possible strategies.

Denote \( \mathcal{M} = \{ \hat{a}^{(m)}(s), \forall m \in \{1, 2, \ldots, M\} \} \) to be the set of all possible strategies, i.e., the *strategy space*.

*Remark.* Enumerating the entire strategy space may be intractable.
Charactering Strategy Space

Optimal decision rule is monotonic:

**Theorem**

*Given two arbitrary channel states \( k, l \in S \) such that \( k < l \), then the optimal strategy for any SA \( i \) satisfies \( \hat{a}_i(k) \leq \hat{a}_i(l), \forall i \in \mathcal{N} \).*

In other words, the optimal strategy has the following threshold form:

\[
\hat{a}_i(s_i(t)) = \begin{cases} 
1, & \text{if } s_i(t) > s_i^* \\
0, & \text{otherwise}
\end{cases}
\]

for some threshold \( s_i^* \in S \).

*Remark.* The number of effective strategies is reduced to \( \tilde{M} = \prod_{i=1}^{N} |S| \).
We assume that all SAs will receive a *delayed* feedback from the remote server about the sensing actions, achieved information utility and energy consumption.

The energy queue is updated based on the delayed information.

\[
Q(t + 1) = \max[Q(t) + p(t - D) - C, 0],
\]  

(20)
Distributed Solution (D-IDDS)

Define the $D$-slot conditional Lyapunov drift as:

$$\Delta(Q(t + D)) = L(t + D + 1) - L(t + D).$$  \hspace{1cm} (21)

In a distributed setting, we try to minimize the upper bound of the modified drift-minus-utility function:

$$\min \sup \Delta(Q(t + D)) - \mathbb{E}\{u(t)|Q(t)\}.$$  \hspace{1cm} (22)

**Lemma**

*Under arbitrary control decisions, the following inequality holds*

$$\Delta(Q(t + D)) - \mathbb{E}\{u(t)|Q(t)\} \leq B(1 + 2D) - CQ(t) + Q(t)p(t) - Vu(t)$$  \hspace{1cm} (23)

where $B = \frac{1}{2}C^2$ is a finite constant.
Distributed Solution (D-IDDS)

\[ \min_{a_i(t), \forall i \in \mathcal{N}} Q(t)p(t) - Vu(t) \quad \text{(24)} \]
\[ \text{s.t.} 
\quad a_i(t) \in \{0, 1\}. \quad \text{(25)} \]

Remark. In a distributed setting, each SA $i$ does not have the information of the channel states and actions at the other SAs. Therefore, $p(t)$ and $u(t)$ are unknown to individual SAs.

Estimate $p(t)$ and $u(t)$ under strategy $\hat{a}^{(m)}(s)$ from the past feedback:

\[ \hat{p}^{(m)}(t) = \frac{1}{W} \sum_{w=1}^{W} \sum_{i \in \mathcal{N}} \hat{p}_i(\hat{a}_i^{(m)}(s_i(t - D - w)), s_i(t - D - w)) \quad \text{(26)} \]
\[ \hat{u}^{(m)}(t) = \frac{1}{W} \sum_{w=1}^{W} \hat{u}(\hat{a}^{(m)}(s(t - D - w))) \quad \text{(27)} \]

where $W > 0$ is the learning horizon.
Remark.

- Based the delayed feedback about $s(t - D), a(t - D), p(t - D)$ and $u(t - D)$, each SA can estimate $\tilde{p}^{(m)}(t)$ and $\tilde{u}^{(m)}(t)$.
- Each SA chooses the pure strategy function $\hat{a}^{(m)}(s)$ from $\tilde{\mathcal{M}}$ that minimizes $Q(t)\tilde{p}^{(m)}(t) - V\tilde{u}^{(m)}(t)$;
Theoretical Analysis

Theorem (C-IDDS System-wide Utility Optimality)

- The gap between the optimal utility and the utility achieved by C-IDDS is upper-bounded:
  \[
  \bar{u}^{opt} - \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[u(t)] \leq \frac{B}{V}
  \]  
  (28)

- The proposed algorithm can guarantee the time average energy consumption to be upper-bounded:
  \[
  \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\rho(t)] \leq C + O\left(\sqrt{\frac{V}{T}}\right).
  \]  
  (29)
Theoretical Analysis

Theorem (D-IDDS System-wide Utility Optimality)

- The gap between the optimal utility and the utility achieved by D-IDDS is upper-bounded:

\[
\bar{u}^{opt} - \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[u(t)] \leq \frac{B(1 + 2D)}{V} + \frac{\mathbb{E}(L(D))}{VT} + O\left(\frac{1}{\sqrt{W}}\right),
\]

(30)

- The proposed algorithm can guarantee the time average energy consumption to be upper-bounded:

\[
\frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[p(t)] \leq C + O\left(\sqrt{\frac{V}{T}}\right).
\]

(31)
Deployed Testbed

Figure: Developed *EA-Hub* prototype device.

Figure: Deployed testbed comprising 23 EA-Hub devices.
Experiment Setup

Objective: collaboratively estimate the average room temperature denoted by $\theta$.

Assuming a Gaussian model for the prior distribution of $\theta$: $\theta \sim N\left(\frac{\omega}{\nu}, \frac{1}{\nu}\right)$.

When a new batch of measurements $z$ are reported by the sensors, the hyperparameter can be updated as:

$$
\omega \leftarrow \omega + \sum_{z_i \in z} z_i, \quad (32)
$$

$$
\nu \leftarrow \nu + |z|.
$$

The information utility brought by the measurements $z$ is calculated as the KL-Divergence between the prior and posterior distributions according to (7).
Average energy consumption constraint: $C = 10$.

**Figure:** Estimation error vs time

**Figure:** Average energy consumption vs time.

**Figure:** Reporting frequency vs time.
**Experimental Results**

![Graphs showing average information utility vs V.](image1)

![Graphs showing average energy consumption vs V.](image2)

![Graphs showing average information utility vs C.](image3)

**Figure:** Average information utility vs $V$.

**Figure:** Average energy consumption vs $V$.

**Figure:** Average information utility vs $C$.

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We proposed C-IDDS and D-IDDS, a centralized and a distributed algorithm for the sensing agents to make optimal sensing decisions for efficient Bayesian inference with exponential family distributions.

Both C-IDDS and D-IDDS are online algorithms which can adapt to stochastic system conditions without any future information.

Through rigorous theoretical analysis, we prove that the proposed algorithms can achieve an asymptotically optimal system-wide utility.

A real testbed has been built to evaluate the performance of the proposed algorithms in real-world environments.
Thanks.

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